KEY STAGE 2		
In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.		
Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping, bar model		
Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply. In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.	**Multiplication and division:** Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.	**Fractions:** Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside. In Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1. Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

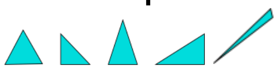
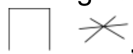
The following pages show the *Power Maths/HVPA* progression in calculation (addition, subtraction, multiplication and division) and how this works in line with the National Curriculum. The consistent use of the CPA (concrete, pictorial, abstract) approach across our curriculum helps children develop mastery across all the operations in an efficient and reliable way. This policy shows how these methods develop children's confidence in their understanding of both written and mental methods.

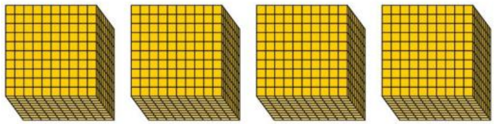
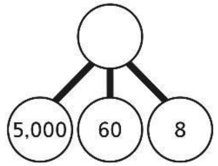
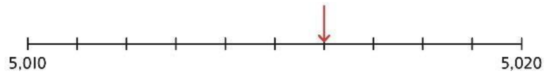
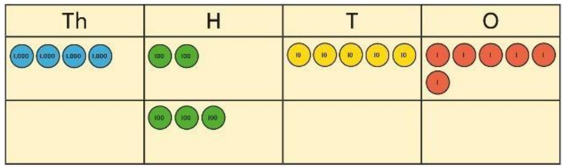
The **CONCRETE** stage introduces real objects that children can use to 'do' the maths − any familiar object that a child can manipulate and move to help bring the maths to life. It is important to appreciate, however, that children must always understand the link between models and the objects they represent. Although they can be used at any time, good concrete models are an essential first step in understanding.

PICTORIAL representations of objects to let children 'see' what particular maths problems look like. It helps them make connections between the concrete and pictorial representations and the abstract maths concept. Children can also create or view a pictorial representation together, enabling discussion and comparisons.

Our ultimate goal is for children to understand **ABSTRACT** mathematical concepts, signs and notation and, of course, some children will reach this stage far more quickly than others. To work with abstract concepts, a child needs to be comfortable with the meaning of, and relationships between, concrete, pictorial and abstract models and representations. The C-P-A approach is not linear, and children may need different types of models at different times. However, when a child demonstrates with concrete models and pictorial representations that they have grasped a concept, we can be confident that they are ready to explore or model it with abstract signs such as numbers and notation

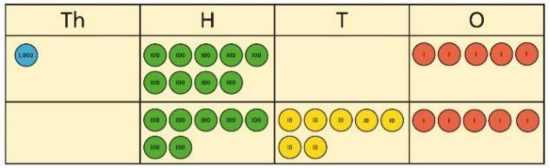
What are Non-examples?

Even with  as examples, a learner does not have enough information to know what is not a triangle. Selected non-examples, like, , help focus attention on details that might otherwise be missed. The "three sides" must be straight, not curved; there can be no extra frills or bows or hanging-over bits of line (line segments must intersect only at their endpoints); the "points" can't be "cut off" (the shape is bounded by only three segments); the figure must be closed (all endpoints must be joined).
These non-examples were selected to be "near-misses," very close to the image people have of triangles. When children give verbal descriptions of triangles, they often mention "three lines" or "three corners," but omit the details that eliminate even fairly distant misses, like, , which may sometimes be useful non-examples to help children improve their verbal descriptions.

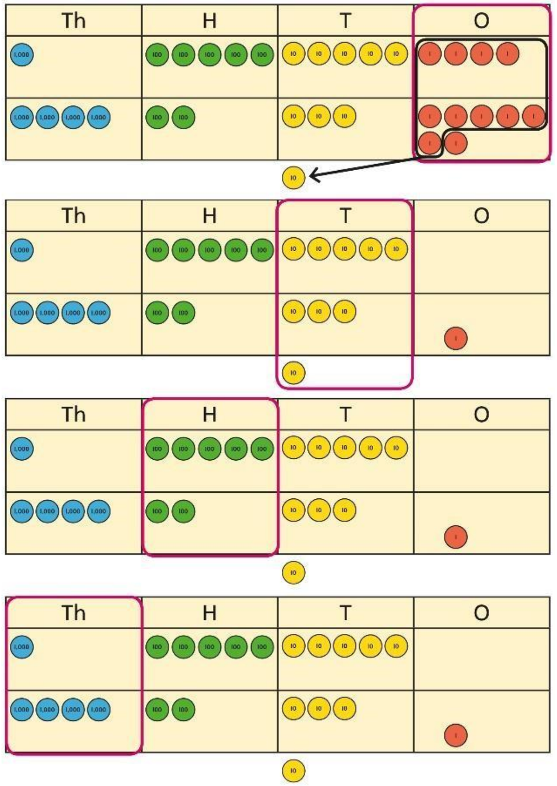
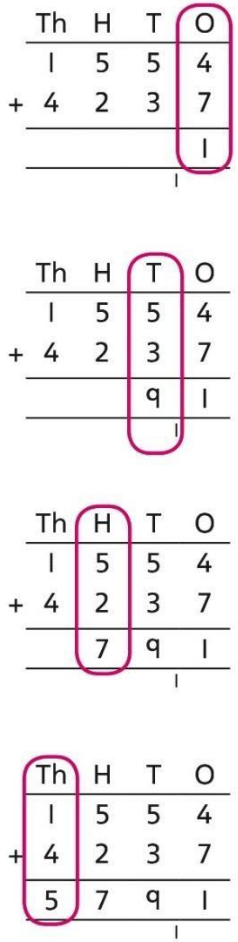
Year 4			
Year 4 Addition	**Concrete**	**Pictorial**	**Abstract**
Understanding numbers to 10,000	Use place value equipment to understand the place value of 4-digit numbers.  *4 thousands equal 4,000.* *1 thousand is 10 hundreds.*	Represent numbers using place value counters once children understand the relationship between 1,000s and 100s.  *2,000 + 500 + 40 + 2 = 2,542*	Understand partitioning of 4-digit numbers, including numbers with digits of 0.  *5,000 + 60 + 8 = 5,068* Understand and read 4-digit numbers on a number line. 
Choosing mental methods where appropriate	Use unitising and known facts to support mental calculations. *Make 1,405 from place value equipment.* *Add 2,000.* *Now add the 1,000s.* *1 thousand + 2 thousands = 3 thousands* *1,405 + 2,000 = 3,405*	Use unitising and known facts to support mental calculations.  *I can add the 100s mentally.* *200 + 300 = 500* *So, 4,256 + 300 = 4,556*	Use unitising and known facts to support mental calculations. *4,256 + 300 = ?* *2 + 3 = 5* *200 + 300 = 500* *4,256 + 300 = 4,556*

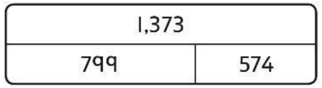
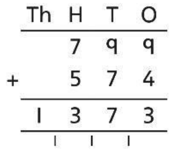
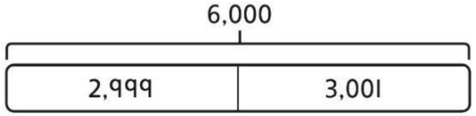
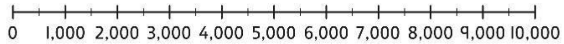
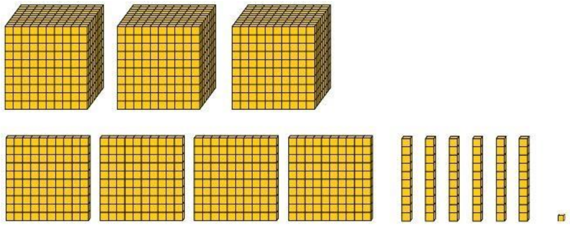
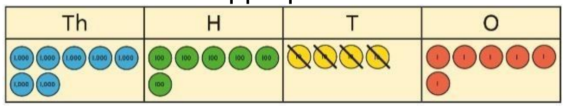
<table>
<tr>
<td>Column addition with exchange</td>
<td>Use place value equipment on a place value grid to organise thinking.

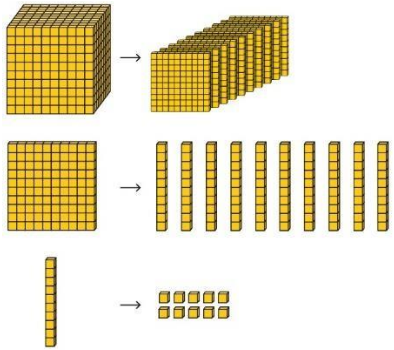
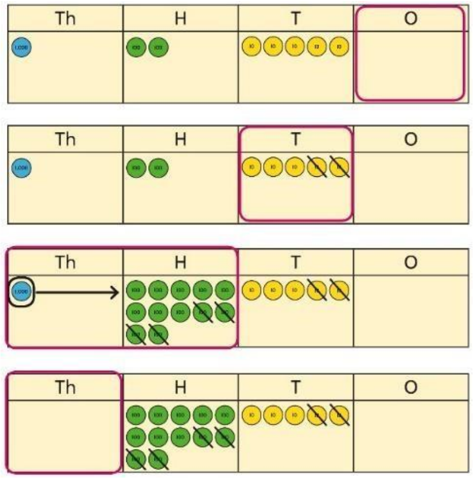
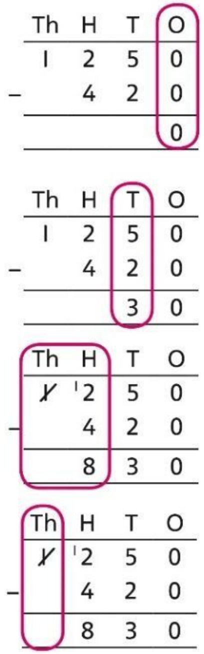
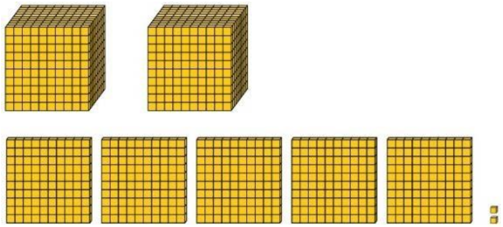
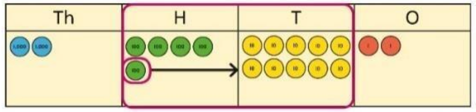
Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.

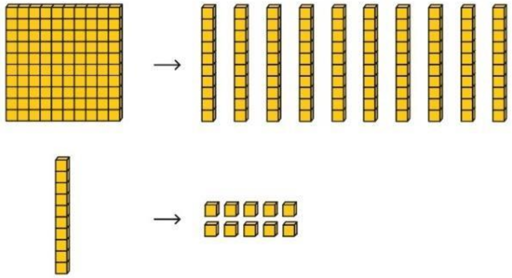
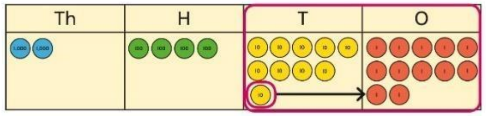
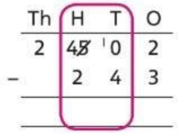
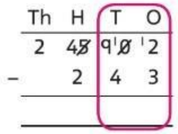
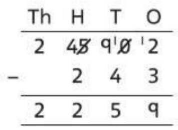
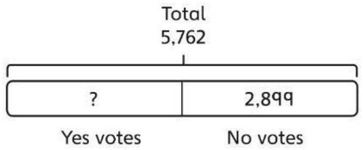
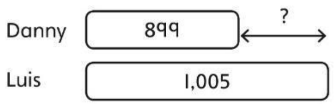
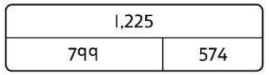
Use equipment.to show 1,905 + 775.


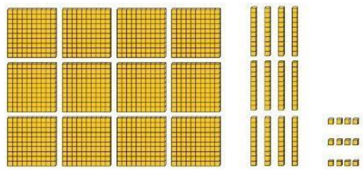
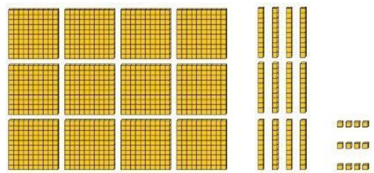
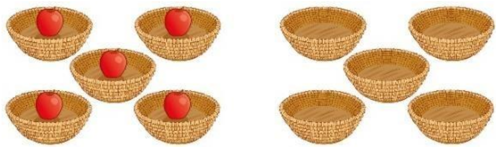
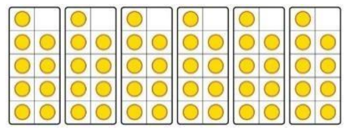
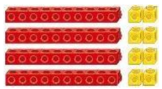
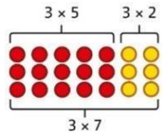
Why have only three columns been used for the second row? Why is the Thousands box empty?

Which columns will total 10 or more?</td>
<td>Use place value equipment to model required exchanges.

Include examples that exchange in more than one column.</td>
<td>Use a column method to add, including exchanges.

Include examples that exchange in more than one column.</td>
</tr>
</table>

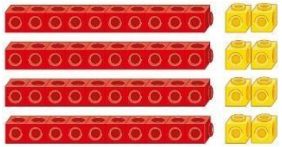
Representing additions and checking strategies		Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.   *I chose to work out 574 + 800, then subtract 1.*  *This is equivalent to 3,000 + 3,000.*	Use rounding and estimating on a number line to check the reasonableness of an addition.  *912 + 6,149 = ?* *I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.*
Year 4 Subtraction	**Concrete**	**Pictorial**	**Abstract**
Choosing mental methods where appropriate	Use place value equipment to justify mental methods.  *What number will be left if we take away 300?*	Use place value grids to support mental methods where appropriate.  *7,646 − 40 = 7,606*	Use knowledge of place value and unitising to subtract mentally where appropriate. *3,501 − 2,000* *3 thousands − 2 thousands = 1 thousand* *3,501 − 2,000 = 1,501*

Column subtraction with exchange	Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary. 	Represent place value equipment on a place value grid to subtract, including exchanges where needed. 	Use column subtraction, with understanding of the place value of any exchange required. 
Column subtraction with exchange across more than one column	Understand why two exchanges may be necessary. *2,502 − 243 = ?* 	Make exchanges across more than one column where there is a zero as a place holder. *2,502 − 243 = ?* 	Make exchanges across more than one column where there is a zero as a place holder. *2,502 − 243 = ?*

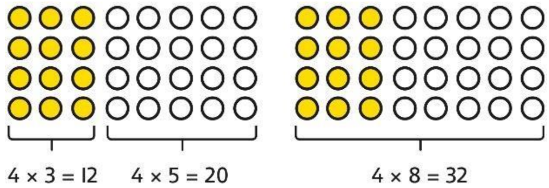
	I need to exchange a 10 for some 1s, but there are not any 10s here. 	 	  
Representing subtractions and checking strategies		Use bar models to represent subtractions where a part needs to be calculated.  *I can work out the total number of Yes votes using 5,762 − 2,899.* Bar models can also represent ‘find the difference’ as a subtraction problem.  	Use inverse operations to check subtractions. *I calculated 1,225 − 799 = 574.* *I will check by adding the parts.*   *The parts do not add to make 1,225.* *I must have made a mistake.*

Year 4 Multiplication	**Concrete**	**Pictorial**	**Abstract**
Multiplying by multiples of 10 and 100	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds is 12 hundreds.*	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  $3 \times 4 = 12$ $3 \times 40 = 120$ $3 \times 400 = 1{,}200$	Use known facts and understanding of place value and commutativity to multiply mentally. $4 \times 7 = 28$ $4 \times 70 = 280$ $40 \times 7 = 280$ $4 \times 700 = 2{,}800$ $400 \times 7 = 2{,}800$
Understanding times-tables up to 12 × 12	Understand the special cases of multiplying by 1 and 0.  $5 \times 1 = 5$ $\quad$ $5 \times 0 = 0$	Represent the relationship between the ×9 table and the ×10 table.  Represent the ×11 table and ×12 tables in relation to the ×10 table.  $2 \times 11 = 20 + 2$ $3 \times 11 = 30 + 3$ $4 \times 11 = 40 + 4$  $4 \times 12 = 40 + 8$	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table *5 × 6 is double 5 × 3* ×5 table and ×6 table *I know that 7 × 5 = 35 so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table $3 \times 7 = 3 \times 5 + 3 \times 2$  ×9 table and ×10 table $6 \times 10 = 60$ $6 \times 9 \ \ = 60 - 6$

<table>
<tr>
<td>Understanding and using partitioning in multiplication</td>
<td>Make multiplications by partitioning.

4 × 12 is 4 groups of 10 and 4 groups of 2.


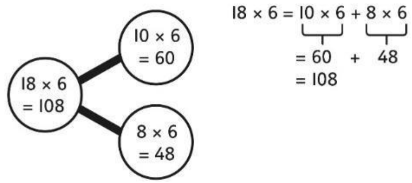
4 × 12 = 40 + 8</td>
<td>Understand how multiplication and partitioning are related through addition.



4 × 3 = 12
4 × 5 = 20
12 + 20 = 32

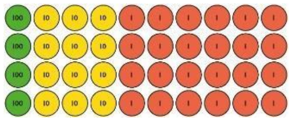
4 × 8 = 32</td>
<td>Use partitioning to multiply 2-digit numbers by a single digit.

18 × 6 = ?



18 × 6 = 10 × 6 + 8 × 6
= 60 + 48
= 108</td>
</tr>
<tr>
<td>Column multiplication for 2- and 3-digit numbers multiplied by a single digit</td>
<td>Use place value equipment to make multiplications.

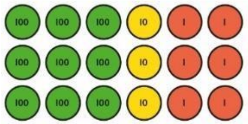
Make 4 × 136 using equipment.



I can work out how many 1s, 10s and 100s.

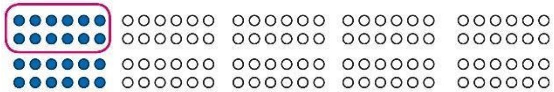
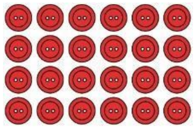
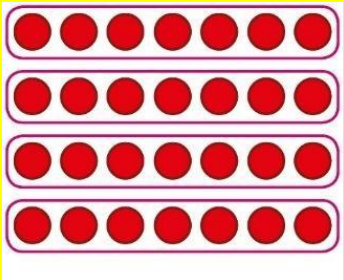
There are 4 × 6 ones… 24 ones
There are 4 × 3 tens … 12 tens
There are 4 × 1 hundreds … 4 hundreds

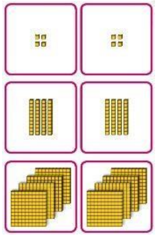
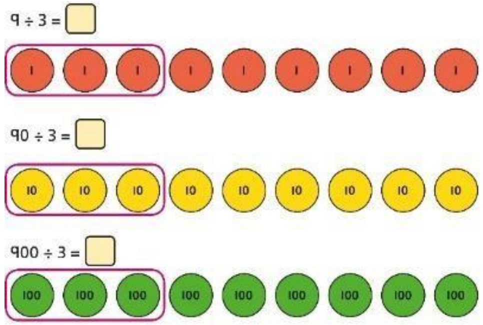
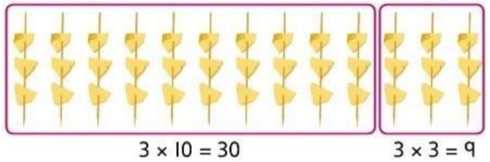
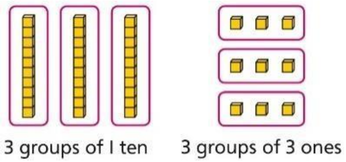
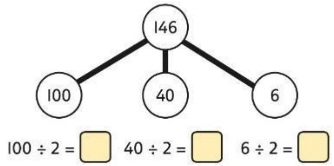
24 + 120 + 400 = 544</td>
<td>Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.

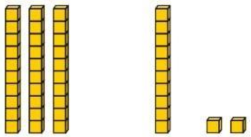
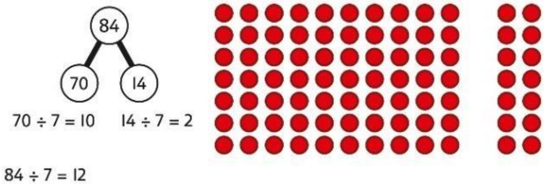
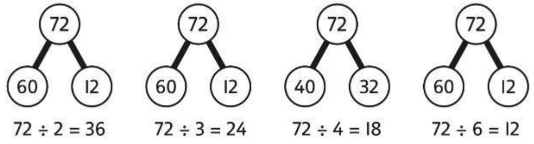
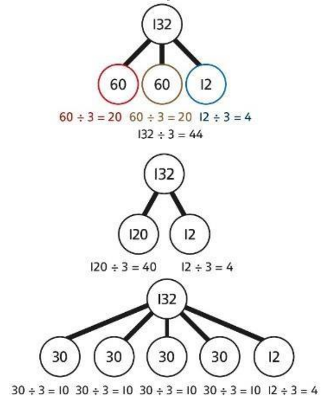
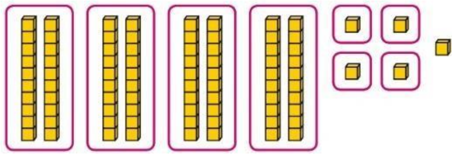
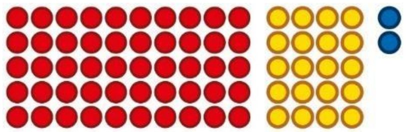
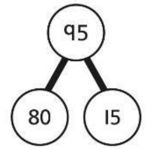

$\begin{array}{r} 3\ 1\ 2 \\ \times\quad\ 3 \\ \hline 9\ 3\ 6 \\ \hline \end{array}$</td>
<td>Use the formal column method for up to 3-digit numbers multiplied by a single digit.
$\begin{array}{r} 3\ 1\ 2 \\ \times\quad\ 3 \\ \hline 9\ 3\ 6 \\ \hline \end{array}$

Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation.

$\begin{array}{r} 2\ 3 \\ \times\ \ 5 \\ \hline 1\ 5 \\ 1\ 0\ 0 \\ \hline 1\ 1\ 5 \\ \hline \end{array}$ $\begin{array}{r} 2\ 3 \\ \times\ \ 5 \\ \hline 1\ 1\ 5 \\ \hline {}_{1}\ \ \end{array}$</td>
</tr>
</table>

Multiplying more than two numbers	Represent situations by multiplying three numbers together.  *Each sheet has 2 × 5 stickers.* *There are 3 sheets.* *There are 5 × 2 × 3 stickers in total.* $\underbrace{5 \times 2}_{} \times 3 = 30$ $10 \times 3 = 30$	Understand that commutativity can be used to multiply in different orders.  *2 × 6 × 10 = 120* *12 × 10 = 120* *10 × 6 × 2 = 120* *60 × 2 = 120*	Use knowledge of factors to simplify some multiplications. *24 × 5 = 12 × 2 × 5* $12 \times \underbrace{2 \times 5}_{} =$ $12 \times 10 = 120$ So, 24 × 5 = 120
Year 4 **Division**	**Concrete**	**Pictorial**	**Abstract**
Understanding the relationship between multiplication and division, including times-tables	Use objects to explore families of multiplication and division facts.  *4 × 6 = 24* *24 is 6 groups of 4.* *24 is 4 groups of 6.* *24 divided by 6 is 4.* *24 divided by 4 is 6.*	Represent divisions using an array.  28 ÷ 7 = 4	Understand families of related multiplication and division facts. *I know that 5 × 7 = 35* *so I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5* *35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* *5 = 35 ÷ 7*

Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide.  *8 ones divided into 2 equal groups* *4 ones in each group* *8 tens divided into 2 equal groups* *4 tens in each group* *8 hundreds divided into 2 equal groups* *4 hundreds in each group*	Represent divisions using place value equipment.  *9 ÷ 3 = 3* *9 tens divided by 3 is 3 tens.* *9 hundreds divided by 3 is 3 hundreds.*	Use known facts to divide 10s and 100s by a single digit. *15 ÷ 3 = 5* *150 ÷ 3 = 50* *1500 ÷ 3 = 500*
Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	Partition into 10s and 1s to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. *142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* *142 ÷ 2 = 73*

Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed. *42 ÷ 3 = ?* *I will split it into 30 and 12, so that I can divide by 3 more easily.* 	Represent how to partition flexibly where needed. *84 ÷ 7 = ?* *I will partition into 70 and 14 because I am dividing by 7.*  	Make decisions about appropriate partitioning based on the division required.  Understand that different partitions can be used to complete the same division. 
Understanding remainders	Use place value equipment to find remainders. *85 shared into 4 equal groups* *There are 24, and 1 that cannot be shared.* 	Represent the remainder as the part that cannot be shared equally.  *72 ÷ 5 = 14 remainder 2*	Understand how partitioning can reveal remainders of divisions.  *80 ÷ 4 = 20* *12 ÷ 4 = 3* *95 ÷ 4 = 23 remainder 3*