



## KEY STAGE 2

In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

**Key language:** decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number

**Addition and subtraction:** Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage.

Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods.

Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

**Multiplication and division:** Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers.

Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000.

Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions.

Multiplication and division of decimals are also introduced and refined in Year 6.

**Fractions:** Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic.

Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.

The following pages show the *Power Maths/HVPA* progression in calculation (addition, subtraction, multiplication and division) and how this works in line with the National Curriculum. The consistent use of the CPA (concrete, pictorial, abstract) approach across our curriculum helps children develop mastery across all the operations in an efficient and reliable way. This policy shows how these methods develop children’s confidence in their understanding of both written and mental methods.

The **CONCRETE** stage introduces real objects that children can use to ‘do’ the maths – any familiar object that a child can manipulate and move to help bring the maths to life. It is important to appreciate, however, that children must always understand the link between models and the objects they represent. Although they can be used at any time, good concrete models are an essential first step in understanding.

**PICTORIAL** representations of objects to let children ‘see’ what particular maths problems look like. It helps them make connections between the concrete and pictorial representations and the abstract maths concept. Children can also create or view a pictorial representation together, enabling discussion and comparisons.

Our ultimate goal is for children to understand **ABSTRACT** mathematical concepts, signs and notation and, of course, some children will reach this stage far more quickly than others. To work with abstract concepts, a child needs to be comfortable with the meaning of, and relationships between, concrete, pictorial and abstract models and representations. The C-P-A approach is not linear, and children may need different types of models at different times. However, when a child demonstrates with concrete models and pictorial representations that they have grasped a concept, we can be confident that they are ready to explore or model it with abstract signs such as numbers and notation.

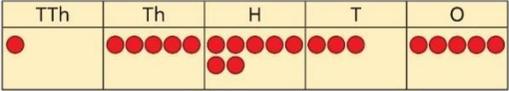
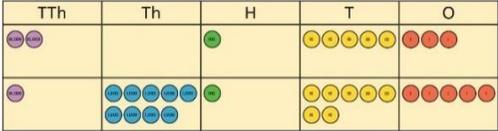
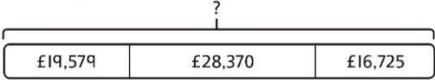
### What are Non-examples?

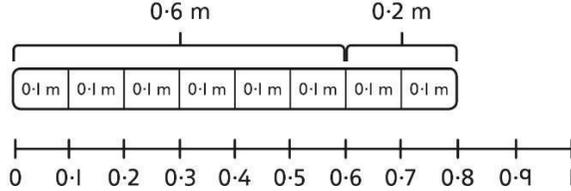
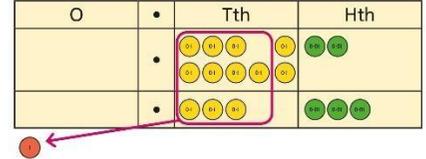
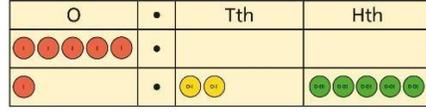
Even with  as examples, a learner does not have enough information to know what is not a triangle. Selected non-examples,

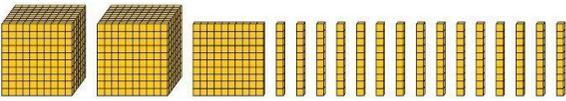
like, , help focus attention on details that might otherwise be missed. The “three sides” must be straight, not curved; there can be no extra frills or bows or hanging-over bits of line (line segments must intersect only at their endpoints); the “points” can’t be “cut off” (the shape is bounded by only three segments); the figure must be closed (all endpoints must be joined).

These non-examples were selected to be “near-misses,” very close to the image people have of triangles. When children give verbal descriptions of triangles, they often mention “three lines” or “three corners,” but omit the details that eliminate even fairly distant misses, like,  , which may sometimes be useful non-examples to help children improve their verbal descriptions.

Year 5

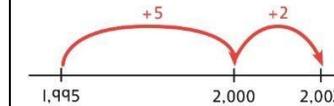
Year 5 Addition	Concrete →	Pictorial →	Abstract
<p><b>Column addition with whole numbers</b></p>	<p>Use place value equipment to represent additions.</p> <p><i>Add a row of counters onto the place value grid to show 15,735 + 4,012.</i></p> 	<p>Represent additions, using place value equipment on a place value grid alongside written methods.</p>  <p><i>I need to exchange 10 tens for a 100.</i></p> $\begin{array}{r} \text{TTh Th H T O} \\ 20153 \\ + 19175 \\ \hline 39328 \end{array}$	<p>Use column addition, including exchanges.</p> $\begin{array}{r} \text{TTh Th H T O} \\ 19175 \\ + 18417 \\ \hline 37592 \end{array}$
<p><b>Representing additions</b></p>		<p>Bar models represent addition of two or more numbers in the context of problem solving.</p>  <p>Jen: £2,600</p> <p>Holly: £2,600 and £1,450</p> <p>A bracket on the right side of the two boxes is labeled with a question mark.</p> <p>£4,050</p> $\begin{array}{r} \text{Th H T O} \\ 2600 \\ + 1450 \\ \hline 4050 \end{array}$ $\begin{array}{r} \text{Th H T O} \\ 2600 \\ + 4050 \\ \hline 6650 \end{array}$	<p>Use approximation to check whether answers are reasonable.</p> $\begin{array}{r} \text{TTh Th H T O} \\ 23405 \\ + 7892 \\ \hline 20297 \end{array}$ $\begin{array}{r} \text{TTh Th H T O} \\ 23405 \\ + 7892 \\ \hline 31297 \end{array}$ <p><i>I will use 23,000 + 8,000 to check.</i></p>

<p><b>Adding tenths</b></p>	<p>Link measure with addition of decimals.</p> <p><i>Two lengths of fencing are 0.6 m and 0.2 m. How long are they when added together?</i></p> 	<p>Use a bar model with a number line to add tenths.</p>  <p><math>0.6 + 0.2 = 0.8</math> <i>6 tenths + 2 tenths = 8 tenths</i></p>	<p>Understand the link with adding fractions.</p> $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ <p><i>6 tenths + 2 tenths = 8 tenths</i> <math>0.6 + 0.2 = 0.8</math></p>
<p><b>Adding decimals using column addition</b></p>	<p>Use place value equipment to represent additions.</p> <p><i>Show <math>0.23 + 0.45</math> using place value counters.</i></p>	<p>Use place value equipment on a place value grid to represent additions.</p> <p>Represent exchange where necessary.</p>  $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 0 \cdot 2 \ 3 \\ + 0 \cdot 4 \ 5 \\ \hline 0 \cdot 6 \ 8 \end{array}$ <p>Include examples where the numbers of decimal places are different.</p>  $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 5 \cdot 0 \ 0 \\ + 1 \cdot 2 \ 5 \\ \hline 6 \cdot 2 \ 5 \end{array}$	<p>Add using a column method, ensuring that children understand the link with place value.</p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 0 \cdot 2 \ 3 \\ + 0 \cdot 4 \ 5 \\ \hline 0 \cdot 6 \ 8 \end{array}$ <p>Include exchange where required, alongside an understanding of place value.</p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 0 \cdot 4 \ 2 \\ + 0 \cdot 3 \ 3 \\ \hline 1 \cdot 2 \ 5 \end{array}$ <p>Include additions where the numbers of decimal places are different.</p> <p><math>3.4 + 0.65 = ?</math></p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 3 \cdot 4 \ 0 \\ + 0 \cdot 6 \ 5 \\ \hline \end{array}$

Year 5 Subtraction	Concrete →	Pictorial →	Abstract																																																																																																														
<p><b>Column subtraction with whole numbers</b></p>	<p>Use place value equipment to understand where exchanges are required.</p> <p><math>2,250 - 1,070</math></p> 	<p>Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.</p> <p><math>15,735 - 2,582 = 13,153</math></p> <table border="1" data-bbox="965 464 1543 564"> <thead> <tr> <th>TTh</th> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> <th></th> </tr> </thead> <tbody> <tr> <td>●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>TTh Th H T O</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>●●●●</td> <td>1 5 7 3 5</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>●●●●</td> <td>- 2 5 8 2</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>3</td> </tr> </tbody> </table> <p>Now subtract the 10s. Exchange 1 hundred for 10 tens.</p> <table border="1" data-bbox="965 608 1543 708"> <thead> <tr> <th>TTh</th> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> <th></th> </tr> </thead> <tbody> <tr> <td>●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>TTh Th H T O</td> </tr> <tr> <td></td> <td></td> <td>●●</td> <td>●●●●●●●●</td> <td>●●●●</td> <td>1 5 7 3 5</td> </tr> <tr> <td></td> <td></td> <td>●●</td> <td>●●●●●●●●</td> <td>●●●●</td> <td>- 2 5 8 2</td> </tr> <tr> <td></td> <td></td> <td></td> <td>●●●●</td> <td></td> <td>5 3</td> </tr> </tbody> </table> <p>Subtract the 100s, 1,000s and 10,000s.</p> <table border="1" data-bbox="965 735 1543 836"> <thead> <tr> <th>TTh</th> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> <th></th> </tr> </thead> <tbody> <tr> <td>●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>●●●●●●●●</td> <td>TTh Th H T O</td> </tr> <tr> <td></td> <td>●●</td> <td>●●●●</td> <td>●●●●</td> <td>●●●●</td> <td>1 5 7 3 5</td> </tr> <tr> <td></td> <td>●●</td> <td>●●●●</td> <td>●●●●</td> <td>●●●●</td> <td>- 2 5 8 2</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>1 3 1 5 3</td> </tr> </tbody> </table>	TTh	Th	H	T	O		●	●●●●●●●●	●●●●●●●●	●●●●●●●●	●●●●●●●●	TTh Th H T O					●●●●	1 5 7 3 5					●●●●	- 2 5 8 2						3	TTh	Th	H	T	O		●	●●●●●●●●	●●●●●●●●	●●●●●●●●	●●●●●●●●	TTh Th H T O			●●	●●●●●●●●	●●●●	1 5 7 3 5			●●	●●●●●●●●	●●●●	- 2 5 8 2				●●●●		5 3	TTh	Th	H	T	O		●	●●●●●●●●	●●●●●●●●	●●●●●●●●	●●●●●●●●	TTh Th H T O		●●	●●●●	●●●●	●●●●	1 5 7 3 5		●●	●●●●	●●●●	●●●●	- 2 5 8 2						1 3 1 5 3	<p>Use column subtraction methods with exchange where required.</p> <table border="1" data-bbox="1565 325 1771 464"> <thead> <tr> <th>TTh</th> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td><del>5</del></td> <td><del>12</del></td> <td><del>1</del>0</td> <td>9</td> <td>7</td> </tr> <tr> <td>-</td> <td>1</td> <td>8</td> <td>5</td> <td>3</td> </tr> <tr> <td>4</td> <td>3</td> <td>5</td> <td>6</td> <td>3</td> </tr> </tbody> </table> <p><math>62,097 - 18,534 = 43,563</math></p>	TTh	Th	H	T	O	<del>5</del>	<del>12</del>	<del>1</del> 0	9	7	-	1	8	5	3	4	3	5	6	3
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<p><b>Checking strategies and representing subtractions</b></p>		<p>Bar models represent subtractions in problem contexts, including 'find the difference'.</p> <p>Athletics Stadium <span style="border: 1px solid black; padding: 2px 20px;">75,450</span></p> <p>Hockey Centre <span style="border: 1px solid black; padding: 2px 20px;">← 42,300 →</span></p> <p>Velodrome <span style="border: 1px solid black; padding: 2px 20px;">15,735 ← ? →</span></p>	<p>Children can explain the mistake made when the columns have not been ordered correctly.</p> <table border="1" data-bbox="1570 999 1711 1129"> <thead> <tr> <th colspan="5">Bella's working</th> </tr> <tr> <th>TTh</th> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>7</td> <td>8</td> <td>7</td> <td>7</td> </tr> <tr> <td>+</td> <td>4</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>5</td> <td>7</td> <td>9</td> <td>9</td> <td>7</td> </tr> </tbody> </table> <table border="1" data-bbox="1760 999 1901 1129"> <thead> <tr> <th colspan="5">Correct method</th> </tr> <tr> <th>TTh</th> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>7</td> <td>8</td> <td>7</td> <td>7</td> </tr> <tr> <td>+</td> <td>4</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>1</td> <td>8</td> <td>8</td> <td>9</td> </tr> </tbody> </table> <p>Use approximation to check calculations.</p> <p><i>I calculated 18,000 + 4,000 mentally to check my subtraction.</i></p>	Bella's working					TTh	Th	H	T	O	1	7	8	7	7	+	4	0	1	2	5	7	9	9	7	Correct method					TTh	Th	H	T	O	1	7	8	7	7	+	4	0	1	2	2	1	8	8	9																																																												
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**Choosing efficient methods**

To subtract two large numbers that are close, children find the difference by counting on.  $2,002 - 1,995 = ?$



Use addition to check subtractions.  
*I calculated  $7,546 - 2,355 = 5,191$ .  
 I will check using the inverse.*

**Subtracting decimals**

Explore complements to a whole number by working in the context of length.



1 m -  m =  m

$1 - 0.49 = ?$

Use a place value grid to represent the stages of column subtraction, including exchanges where required.

$5.74 - 2.25 = ?$



Exchange 1 tenth for 10 hundredths.



Now subtract the 5 hundredths.

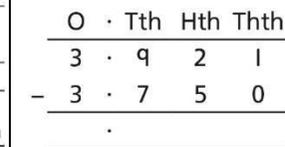


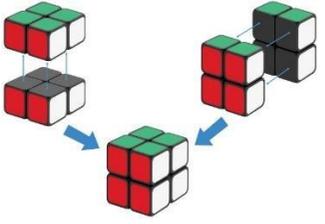
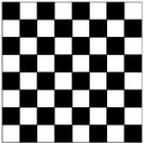
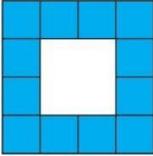
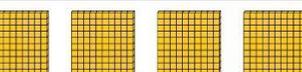
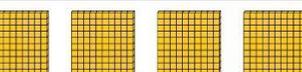
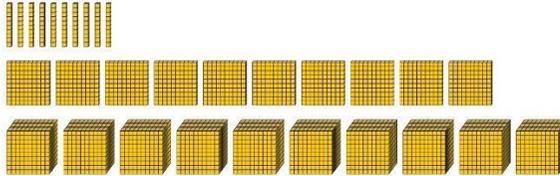
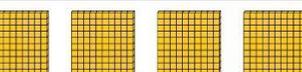
Now subtract the 2 tenths, then the 2 ones.



Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places.

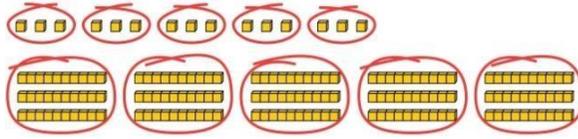
$3.921 - 3.75 = ?$



Year 5 Multiplication	Concrete →	Pictorial →	Abstract												
<p><b>Understanding factors</b></p>	<p>Use cubes or counters to explore the meaning of 'square numbers'.</p> <p><i>25 is a square number because it is made from 5 rows of 5.</i></p> <p>Use cubes to explore cube numbers.</p>  <p><i>8 is a cube number.</i></p>	<p>Use images to explore examples and non-examples of square numbers.</p>  <p><math>8 \times 8 = 64</math> <math>8^2 = 64</math></p>  <p><i>12 is not a square number, because you cannot multiply a whole number by itself to make 12.</i></p>	<p>Understand the pattern of square numbers in the multiplication tables.</p> <p>Use a multiplication grid to circle each square number. Can children spot a pattern?</p>												
<p><b>Multiplying by 10, 100 and 1,000</b></p>	<p>Use place value equipment to multiply by 10, 100 and 1,000 by unitising.</p> <table border="1" data-bbox="358 1005 929 1173"> <tr> <td><math>4 \times 1 = 4 \text{ ones} = 4</math></td> <td></td> </tr> <tr> <td><math>4 \times 10 = 4 \text{ tens} = 40</math></td> <td></td> </tr> <tr> <td><math>4 \times 100 = 4 \text{ hundreds} = 400</math></td> <td></td> </tr> </table>	$4 \times 1 = 4 \text{ ones} = 4$		$4 \times 10 = 4 \text{ tens} = 40$		$4 \times 100 = 4 \text{ hundreds} = 400$		<p>Understand the effect of repeated multiplication by 10.</p> 	<p>Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.</p> <table border="1" data-bbox="1568 1045 1937 1181"> <thead> <tr> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td></td> <td>1</td> <td>7</td> </tr> </tbody> </table> <p><math>17 \times 10 = 170</math>  <math>17 \times 100 = 17 \times 10 \times 10 = 1,700</math>  <math>17 \times 1,000 = 17 \times 10 \times 10 \times 10 = 17,000</math></p>	H	T	O		1	7
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	1	7													

**Multiplying by multiples of 10, 100 and 1,000**

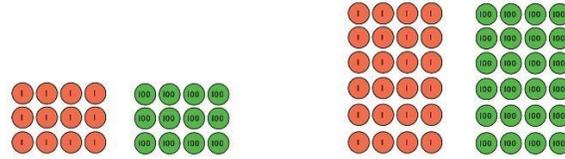
Use place value equipment to explore multiplying by unitising.



5 groups of 3 ones is 15 ones.  
5 groups of 3 tens is 15 tens.

So, I know that 5 groups of 3 thousands would be 15 thousands.

Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.



$4 \times 3 = 12$   
 $4 \times 300 = 1,200$

$6 \times 4 = 24$   
 $6 \times 400 = 2,400$

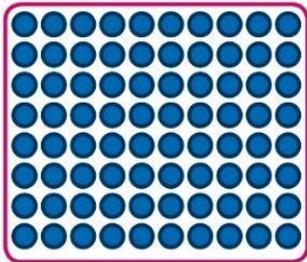
Use known facts and unitising to multiply.

$5 \times 4 = 20$   
 $5 \times 40 = 200$   
 $5 \times 400 = 2,000$   
 $5 \times 4,000 = 20,000$   
 $5,000 \times 4 = 20,000$

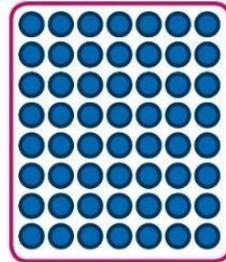
**Multiplying up to 4-digit numbers by a single digit**

Explore how to use partitioning to multiply efficiently.

$8 \times 17 = ?$



$8 \times 10 = 80$



$8 \times 7 = 56$

$80 + 56 = 136$

So,  $8 \times 17 = 136$

Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.

	H	T	O
100		10 10 10 10 10 10	1 1 1
100		10 10 10 10 10 10	1 1 1
100		10 10 10 10 10 10	1 1 1
100		10 10 10 10 10 10	1 1 1
100		10 10 10 10 10 10	1 1 1

Use an area model and then add the parts.

	100	60	3
5	$100 \times 5 = 500$	$60 \times 5 = 300$	$3 \times 5 = 15$

Use a column multiplication, including any required exchanges.

$$\begin{array}{r} 136 \\ \times 5 \\ \hline 680 \\ 150 \\ \hline 685 \end{array}$$

**Multiplying 2-digit numbers by 2-digit numbers**

Partition one number into 10s and 1s, then add the parts.

$23 \times 15 = ?$



$10 \times 15 = 150$



$10 \times 15 = 150$



$3 \times 15 = 45$

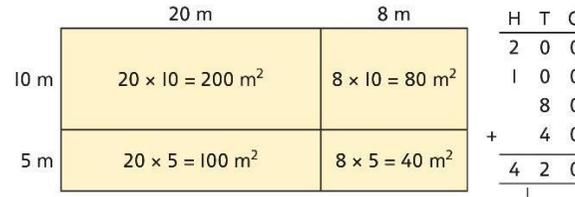
There are 345 bottles of milk in total.

	H	T	O
	1	5	0
	1	5	0
	+	4	5
	3	4	5

$23 \times 15 = 345$

Use an area model and add the parts.

$28 \times 15 = ?$



$28 \times 15 = 420$

Use column multiplication, ensuring understanding of place value at each stage.

	3	4	
	x	2	7
	2	3	8

$34 \times 7$

$34$

$\times 27$

$238$

$34 \times 7$

$680$

$34 \times 20$

$34$

$\times 27$

$238$

$34 \times 7$

$680$

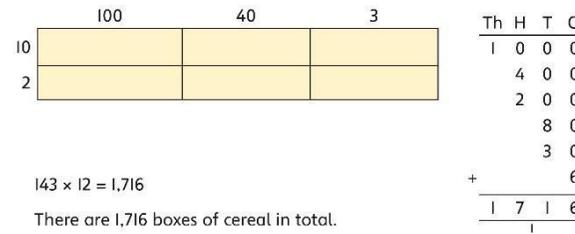
$34 \times 20$

$918$

$34 \times 27$

**Multiplying up to 4-digits by 2-digits**

Use the area model then add the parts.



$143 \times 12 = 1,716$

There are 1,716 boxes of cereal in total.

$143 \times 12 = 1,716$

Use column multiplication, ensuring understanding of place value at each stage.

	1	4	3	
	x	1	2	
	2	8	6	
	1	4	3	0
	1	7	1	6

$143 \times 2$

$143 \times 10$

$143 \times 12$

Progress to include examples that require multiple exchanges as understanding, confidence and fluency build.

$1,274 \times 32 = ?$   
 First multiply 1,274 by 2.

$$\begin{array}{r} 1\ 2\ 7\ 4 \\ \times \quad 3\ 2 \\ \hline 2\ 5\ 4\ 8 \end{array} \quad 1,274 \times 2$$

Then multiply 1,274 by 30.

$$\begin{array}{r} 1\ 2\ 7\ 4 \\ \times \quad 3\ 2 \\ \hline 2\ 5\ 4\ 8 \quad 1,274 \times 2 \\ 3\ 8\ 2\ 2\ 0 \quad 1,274 \times 30 \\ \hline \end{array}$$

Finally, find the total.

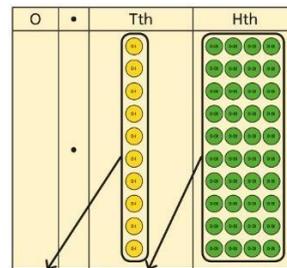
$$\begin{array}{r} 1\ 2\ 7\ 4 \\ \times \quad 3\ 2 \\ \hline 2\ 5\ 4\ 8 \quad 1,274 \times 2 \\ 3\ 8\ 2\ 2\ 0 \quad 1,274 \times 30 \\ \hline 4\ 0\ 7\ 6\ 8 \quad 1,274 \times 32 \end{array}$$

$$\begin{array}{r} 1 \\ 1,274 \times 32 = 40,768 \end{array}$$

**Multiplying decimals by 10, 100 and 1,000**

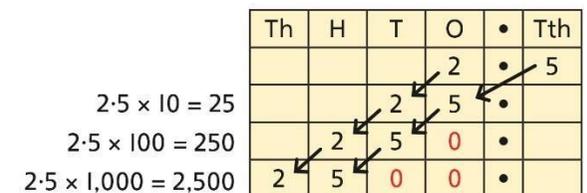
Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.

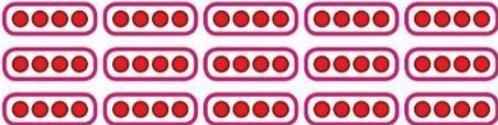
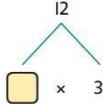
Represent multiplication by 10 as exchange on a place value grid.



$$0.14 \times 10 = 1.4$$

Understand how this exchange is represented on a place value chart.

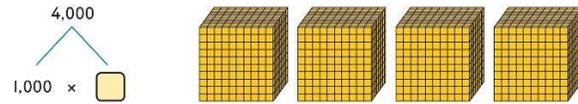


Year 5 Division	Concrete →	Pictorial →	Abstract
<b>Understanding factors and prime numbers</b>	<p>Use equipment to explore the factors of a given number.</p>  <p><math>24 \div 3 = 8</math>  <math>24 \div 8 = 3</math>  <i>8 and 3 are factors of 24 because they divide 24 exactly.</i></p> <p><math>24 \div 5 = 4</math> remainder 4.</p>  <p><i>5 is not a factor of 24 because there is a remainder.</i></p>	<p>Understand that prime numbers are numbers with exactly two factors.</p> <p><math>13 \div 1 = 13</math>  <math>13 \div 2 = 6 \text{ r } 1</math>  <math>13 \div 4 = 4 \text{ r } 1</math></p> <p><i>1 and 13 are the only factors of 13. 13 is a prime number.</i></p>	<p>Understand how to recognise prime and composite numbers.</p> <p><i>I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.</i></p> <p><i>I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.</i></p> <p><i>I know that 1 is not a prime number, as it has only 1 factor.</i></p>
<b>Understanding inverse operations and the link with multiplication, grouping and sharing</b>	<p>Use equipment to group and share and to explore the calculations that are present.</p> <p><i>I have 28 counters.</i></p> <p><i>I made 7 groups of 4. There are 28 in total.</i></p> <p><i>I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.</i></p> <p><i>I have 28 in total. I made groups of 4. There are 7 equal groups.</i></p>	<p>Represent multiplicative relationships and explore the families of division facts.</p>  <p><math>60 \div 4 = 15</math>  <math>60 \div 15 = 4</math></p>	<p>Represent the different multiplicative relationships to solve problems requiring inverse operations.</p> <p><math>12 \div 3 = \square</math>  <math>12 \div \square = 3</math>  <math>\square \times 3 = 12</math>  <math>\square \div 3 = 12</math></p>  <p>Understand missing number problems for division calculations and know how to solve them, using inverse operations.</p> <p><math>22 \div ? = 2</math>  <math>22 \div 2 = ?</math>  <math>? \div 2 = 22</math>  <math>? \div 22 = 2</math></p>

**Dividing whole numbers by 10, 100 and 1,000**

Use place value equipment to support unitising for division.

$$4,000 \div 1,000$$



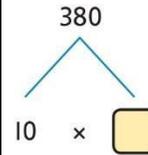
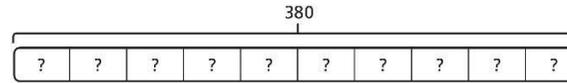
4,000 is 4 thousands.

$$4 \times 1,000 = 4,000$$

So,  $4,000 \div 1,000 = 4$

Use a bar model to support dividing by unitising.

$$380 \div 10 = 38$$



380 is 38 tens.

$$38 \times 10 = 380$$

$$10 \times 38 = 380$$

So,  $380 \div 10 = 38$

Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.

Th	H	T	O
3	2	0	0

$$3,200 \div 100 = ?$$

3,200 is 3 thousands and 2 hundreds.

$$200 \div 100 = 2$$

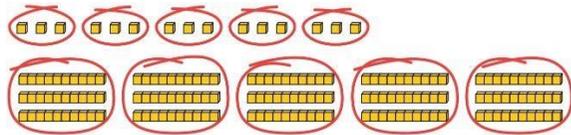
$$3,000 \div 100 = 30$$

$$3,200 \div 100 = 32$$

So, the digits will move two places to the right.

**Dividing by multiples of 10, 100 and 1,000**

Use place value equipment to represent known facts and unitising.



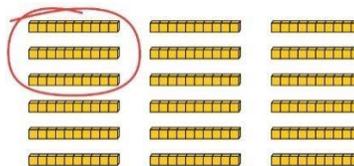
15 ones put into groups of 3 ones. There are 5 groups.

$$15 \div 3 = 5$$

15 tens put into groups of 3 tens. There are 5 groups.

$$150 \div 30 = 5$$

Represent related facts with place value equipment when dividing by unitising.



180 is 18 tens.

18 tens divided into groups of 3 tens. There are 6 groups.

$$180 \div 30 = 6$$

Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check.

$$3,000 \div 5 = 600$$

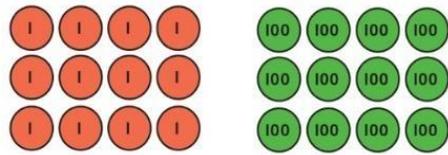
$$3,000 \div 50 = 60$$

$$3,000 \div 500 = 6$$

$$5 \times 600 = 3,000$$

$$50 \times 60 = 3,000$$

$$500 \times 6 = 3,000$$



12 ones divided into groups of 4. There are 3 groups.

12 hundreds divided into groups of 4 hundreds. There are 3 groups.

$$1200 \div 400 = 3$$

**Dividing up to four digits by a single digit using short division**

Explore grouping using place value equipment.

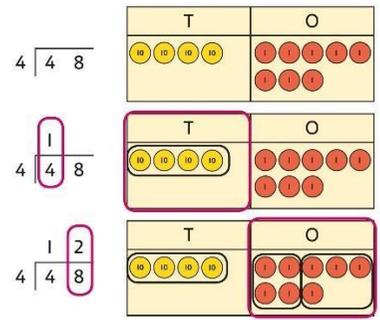
$$268 \div 2 = ?$$

*There is 1 group of 2 hundreds.  
There are 3 groups of 2 tens.  
There are 4 groups of 2 ones.*

$$264 \div 2 = 134$$

Use place value equipment on a place value grid alongside short division.

The model uses grouping. A sharing model can also be used, although the model would need adapting.



Lay out the problem as a short division.

*There is 1 group of 4 in 4 tens.  
There are 2 groups of 4 in 8 ones.*

Use short division for up to 4-digit numbers divided by a single digit.

$$\begin{array}{r} 0 \ 5 \ 5 \ 6 \\ 7 \overline{) 3 \ 8 \ 3 \ 9 \ 4 \ 2} \end{array}$$

$$3,892 \div 7 = 556$$

Use multiplication to check.

$$556 \times 7 = ?$$

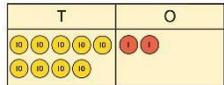
$$6 \times 7 = 42$$

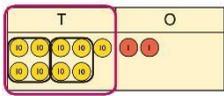
$$50 \times 7 = 350$$

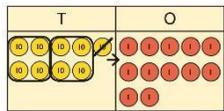
$$500 \times 7 = 3500$$

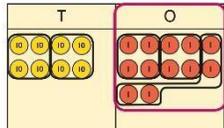
$$3,500 + 350 + 42 = 3,892$$

Work with divisions that require exchange.

$4 \overline{) 92}$ 

 First, lay out the problem.

$4 \overline{) 9} 2$ 

 How many groups of 4 go into 9 tens?  
 2 groups of 4 tens with 1 ten left over.

$4 \overline{) 9} 2$ 

 Exchange the 1 ten left over for 10 ones.  
 We now have 12 ones.

$4 \overline{) 9} 2$ 

 How many groups of 4 go into 12 ones?  
 3 groups of 4 ones.

**Understanding remainders**

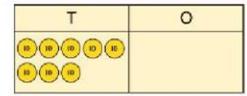
Understand remainders using concrete versions of a problem.

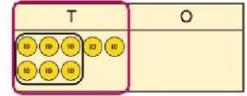
*80 cakes divided into trays of 6.*

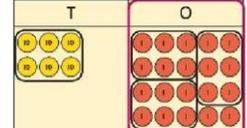


*80 cakes in total. They make 13 groups of 6, with 2 remaining.*

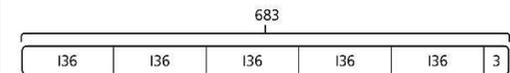
Use short division and understand remainders as the last remaining 1s.

$6 \overline{) 80}$ 

 Lay out the problem as short division.

$6 \overline{) 8} 0$ 

 How many groups of 6 go into 8 tens?  
 There is 1 group of 6 tens.  
 There are 2 tens remaining.

$6 \overline{) 8} 0$ 

 How many groups of 6 go into 20 ones?  
 There are 3 groups of 6 ones.  
 There are 2 ones remaining.

In problem solving contexts, represent divisions including remainders with a bar model.



$683 = 136 \times 5 + 3$   
 $683 \div 5 = 136 \text{ r } 3$

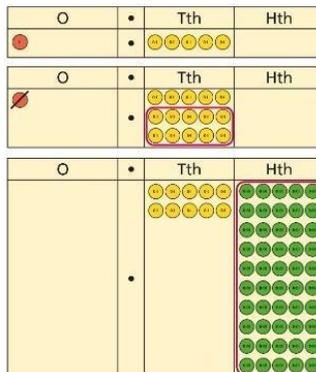
**Dividing decimals by 10, 100 and 1,000**

Understand division by 10 using exchange.

*2 ones are 20 tenths.*

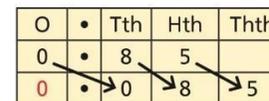
*20 tenths divided by 10 is 2 tenths.*

Represent division using exchange on a place value grid.

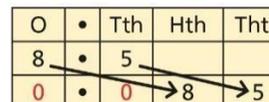


*1.5 is 1 one and 5 tenths. This is equivalent to 10 tenths and 50 hundredths. 10 tenths divided by 10 is 1 tenth. 50 hundredths divided by 10 is 5 hundredths. 1.5 divided by 10 is 1 tenth and 5 hundredths.  $1.5 \div 10 = 0.15$*

Understand the movement of digits on a place value grid.



$0.85 \div 10 = 0.085$

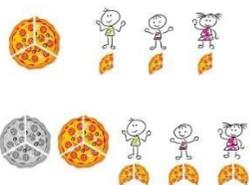


$8.5 \div 100 = 0.085$

**Understanding the relationship between fractions and division**

Use sharing to explore the link between fractions and division.

*1 whole shared between 3 people. Each person receives one-third.*



Use a bar model and other fraction representations to show the link between fractions and division.



$1 \div 3 = \frac{1}{3}$

Use the link between division and fractions to calculate divisions.

$5 \div 4 = \frac{5}{4} = 1 \frac{1}{4}$

$11 \div 4 = \frac{11}{4} = 2 \frac{3}{4}$